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Math 130 - Fall 2025 - Common Final Exam

Print name:____

Section number:______ **Instructor's name:**____________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 200 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- No earbuds, headphones or hoodies may be worn during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	10	10	12	12	16	10	10	12
Score:								

Question:	9	10	11	12	13	14		Total
Points:	16	16	16	18	20	22		200
Score:								

1. (10 points) Find the equation of the tangent line to the graph of

$$y = \frac{3x^2 - 1}{x + 2}$$

at $x = 1$.

Solution: Using the quotient rule,

$$y' = \frac{(6x)(x+2) - (3x^2 - 1)(1)}{(x+2)^2}.$$

At $x = 1$,

$$y(1) = \frac{3(1)^2 - 1}{1 + 2} = \frac{2}{3}, \qquad m = y'(1) = \frac{6(1)(3) - (3 - 1)}{3^2} = \frac{16}{9}.$$

So the tangent line is

$$y - \frac{2}{3} = \frac{16}{9}(x - 1).$$

Equivalently,

$$y = \frac{16}{9}x - \frac{10}{9}.$$

2. (10 points) Solve the equation $8e^{3x-2} = 5e^{x+4}$ using logarithms. Round your final answer to three decimal places.

Solution: Take natural logs after isolating the exponentials:

$$8e^{3x-2} = 5e^{x+4} \implies e^{3x-2} = \frac{5}{8}e^{x+4}.$$

So

$$3x - 2 = \ln\left(\frac{5}{8}\right) + x + 4.$$

Then

$$2x = 6 + \ln\left(\frac{5}{8}\right), \qquad x = 3 + \frac{1}{2}\ln\left(\frac{5}{8}\right).$$

Thus

$$x \approx 2.765.$$

3. A manufacturing company models the total **production cost** (in thousands of dollars) for producing q units of a custom component by the function

$$C(q) = 2q^3e^{-0.5q} + 4q.$$

(a) (8 points) Find the **marginal cost function** $C'(q)$.

Solution: Using the product rule on $2q^3e^{-0.5q}$,

$$C'(q) = 2\left(3q^2e^{-0.5q} + q^3\left(-0.5\right)e^{-0.5q}\right) + 4.$$

So

$$C'(q) = 6q^2e^{-0.5q} - q^3e^{-0.5q} + 4 = e^{-0.5q}(6q^2 - q^3) + 4.$$

Thus,

$$\boxed{C'(q) = e^{-0.5q}(6q^2 - q^3) + 4.}$$

(b) (4 points) Compute the **marginal cost** when $q = 5$ units are produced. Include appropriate units and round your answer to three decimal places.

Solution:

$$C'(5) = e^{-2.5}(6 \cdot 25 - 125) + 4 = 25e^{-2.5} + 4 \approx 6.052.$$

Since C is measured in thousands of dollars, the marginal cost is

$$\boxed{6.052 \text{ thousand dollars per unit}}$$

(about $6,052 per unit).

4. (12 points) Compute the **present value** P of a contract that promises two payments: one of $4,000 three years from now and one of $7,000 six years from now. Assume a continuous annual interest rate of 4.2%.

Solution: For continuous compounding, the present value of a future payment F received in t years is

$$P = Fe^{-rt}.$$

So the total present value is

$$P = 4000e^{-0.042(3)} + 7000e^{-0.042(6)}.$$

Thus

$$P \approx 4000e^{-0.126} + 7000e^{-0.252} \approx 8967.17.$$

Therefore,

$$\boxed{P \approx \$8,967.17}.$$

5. A community theater is studying ticket pricing. The table shows the estimated demand $q = D(p)$ (number of tickets sold) at various price points p (in dollars).

p	8.00	8.50	9.00	9.50	10.00	10.50	11.00
q	1200	1100	1000	920	840	780	730

(a) (4 points) Find the **average rate of change** of demand $q = D(p)$ on the interval $[8, 11]$. Explain the real-world meaning in a sentence, including appropriate units.

Solution:

$$\frac{D(11) - D(8)}{11 - 8} = \frac{730 - 1200}{3} = -\frac{470}{3} \approx -156.667.$$

So the average rate of change is

$$\boxed{-156.667 \text{ tickets per dollar}}.$$

Solution: Over the interval from \$8 to \$11, demand decreases on average by about 156.667 tickets for each \$1 increase in price.

(b) (8 points) **Estimate the elasticity of demand** at the price $p = 10$. You may use any reasonable differences to approximate $D'(10)$ Interpret your result in a sentence.

Solution: Using a central difference,

$$D'(10) \approx \frac{D(10.5) - D(9.5)}{10.5 - 9.5} = \frac{780 - 920}{1} = -140.$$

Elasticity is

$$E(10) = -\frac{10D'(10)}{D(10)} \approx -\frac{10(-140)}{840} = \frac{1400}{840} = \frac{5}{3} \approx 1.667.$$

Thus,

$$\boxed{E(10) \approx 1.667.}$$

Solution: At a price of \$10, the demand has elasticity about 1.667, so a 1% increase in price produces about a 1.667% decrease in demand.

(c) (4 points) At $p = 10$, is demand elastic or inelastic? If price increases slightly from \$10, does **revenue** increase or decrease? Explain briefly.

Solution: Since $E(10) \approx 1.667 > 1$, demand is **elastic**. Therefore, if price increases slightly from \$10, revenue will **decrease**, because the percentage drop in quantity demanded is larger than the percentage rise in price.

6. (10 points) A recent college graduate wants to begin saving for a future home purchase. They plan to deposit a constant amount S dollars continuously each year into an account that earns interest at a continuous rate of 6.4%. How much must be invested per year so that the account will contain $150,000 after 20 years?

Solution: For a continuous income stream invested at rate S dollars per year into an account earning continuous interest rate r, the future value after t years is

$$A = \frac{S}{r}\left(e^{rt} - 1\right).$$

Here $A = 150000$, $r = 0.064$, and $t = 20$, so

$$150000 = \frac{S}{0.064}\left(e^{0.064(20)} - 1\right).$$

Hence

$$S = 150000 \cdot \frac{0.064}{e^{1.28} - 1} \approx 3697.086.$$

Therefore, the required continuous investment rate is

$$\boxed{\$3697.09 \text{ per year}}.$$

7. Let $r(t)$ be the rate of change of the price of a share of ABC Inc. in dollars per day after t days of trading.

   (a) (6 points) Write a sentence with units which gives the meaning of $\int_0^6 r(t)\,dt = -7.5$.

   Solution: The integral means that over the first 6 days, the price of one share of ABC Inc. changed by −$7.50 total. In other words, the share price decreased by $7.50 from day 0 to day 6.

   (b) (4 points) If a share of ABC Inc. cost $25 on day $t = 0$ of trading, use the information above to find its price on day $t = 6$.

   Solution: Since

   $$\int_0^6 r(t)\,dt = \text{change in price} = -7.5,$$

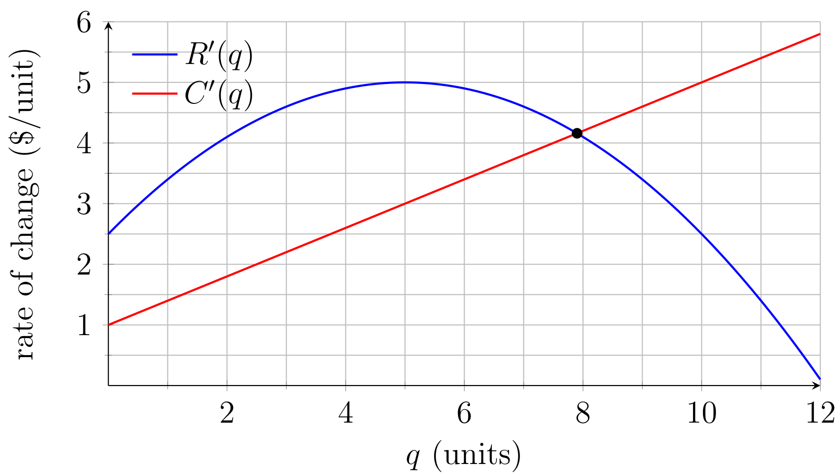
   we have

   $$\text{price at } t = 6 = 25 - 7.5 = 17.5.$$

   So the price on day 6 is

   $$\boxed{\$17.50}.$$

8. The curves below represent the *rates of change* of cost and revenue as functions of quantity q: the **marginal cost** $C'(q) = 1 + 0.4q$ and **marginal revenue** $R'(q) = 5 - 0.1(q-5)^2$.



(a) (4 points) For which values of q does revenue increase faster than cost? (Answer in interval form.)

Solution: Revenue increases faster than cost when

$$R'(q) > C'(q).$$

From the graph, the curves intersect at about $q \approx 7.9$, and on $0 < q < 7.9$ the marginal revenue curve lies above the marginal cost curve. Thus the interval is

$$\boxed{(0, 7.9) \text{ approximately.}}$$

(b) (4 points) Estimate the production level q^* where profit is maximized.

Solution: Profit is maximized where $P'(q) = R'(q) - C'(q) = 0$ and changes from positive to negative, which occurs at the intersection of the two curves. From the graph,

$$\boxed{q^* \approx 7.9 \text{ units}}.$$

(c) (4 points) Estimate the change in profit if production increases from $q = 3$ to $q = 4$.

Solution: The change in profit is approximately

$$\int_3^4 \big(R'(q) - C'(q)\big)\, dq.$$

A reasonable estimate is one unit times the net rate near the middle of the interval. At $q = 3.5$,

$$R'(3.5) = 5 - 0.1(3.5-5)^2 = 4.775, \qquad C'(3.5) = 1 + 0.4(3.5) = 2.4.$$

So

$$\Delta P \approx (4.775 - 2.4)(1) = 2.375.$$

Therefore profit increases by about

$$\boxed{\$2.4 \text{ (approximately)}}.$$

9. Consider the function

$$f(x) = x^3 - 6x^2.$$

(a) (4 points) Find all critical points of $f(x)$ Give the critical point and its function value as an (x, y) pair.

Solution:

$$f'(x) = 3x^2 - 12x = 3x(x-4).$$

So the critical numbers are $x = 0$ and $x = 4$. Their function values are

$$f(0) = 0, \qquad f(4) = 64 - 96 = -32.$$

Thus the critical points are

$$\boxed{(0,0) \text{ and } (4,-32).}$$

(b) (4 points) For each of the critical points found in part (a), determine if it corresponds to a local maximum, local minimum or neither. You may use either the first or second derivative test to justify your answer.

Solution: Use the second derivative:

$$f''(x) = 6x - 12.$$

Then

$$f''(0) = -12 < 0,$$

so $(0,0)$ is a local maximum, and

$$f''(4) = 12 > 0,$$

so $(4,-32)$ is a local minimum.

(c) (4 points) Find the absolute (or global) maximum and absolute (or global) minimum of $f(x)$ on the interval $-1 \leq x \leq 7$.

Solution: Check the endpoints and critical points:

$$f(-1) = -1 - 6 = -7, \qquad f(0) = 0, \qquad f(4) = -32, \qquad f(7) = 343 - 294 = 49.$$

Therefore the absolute minimum is

$$\boxed{-32 \text{ at } x = 4}$$

and the absolute maximum is

$$\boxed{49 \text{ at } x = 7}.$$

(d) (4 points) Find all value(s) of x for which $f''(x) = 0$ and determine whether or not these value(s) are inflection points.

Solution: Since

$$f''(x) = 6x - 12,$$

we solve

$$6x - 12 = 0 \implies x = 2.$$

Because $f''(x) < 0$ for $x < 2$ and $f''(x) > 0$ for $x > 2$, the concavity changes at $x = 2$. Therefore $x = 2$ is an inflection point. Also,

$$f(2) = 8 - 24 = -16,$$

so the inflection point is

$$\boxed{(2, -16).}$$

10. The cost (in thousands of dollars) of producing q tons of a material is

$$C(q) = q^3 - 12q^2 + 60q$$

(a) (4 points) Find a formula for the average cost $a(q)$.

Solution: Average cost is total cost divided by quantity:

$$a(q) = \frac{C(q)}{q} = \frac{q^3 - 12q^2 + 60q}{q} = q^2 - 12q + 60.$$

So

$$\boxed{a(q) = q^2 - 12q + 60.}$$

(b) (4 points) Find a formula for the marginal cost $MC(q)$.

Solution:

$$MC(q) = C'(q) = 3q^2 - 24q + 60.$$

So

$$\boxed{MC(q) = 3q^2 - 24q + 60.}$$

(c) (4 points) Find all production levels where $a(q) = MC(q)$.

Solution: Set the two formulas equal:

$$q^2 - 12q + 60 = 3q^2 - 24q + 60.$$

Then

$$0 = 2q^2 - 12q = 2q(q - 6).$$

So

$$\boxed{q = 0 \text{ or } q = 6.}$$

In context, the meaningful positive production level is $q = 6$.

(d) (4 points) Use the values you found in the previous part to determine the production level which minimizes the average cost.

Solution: Since

$$a(q) = q^2 - 12q + 60,$$

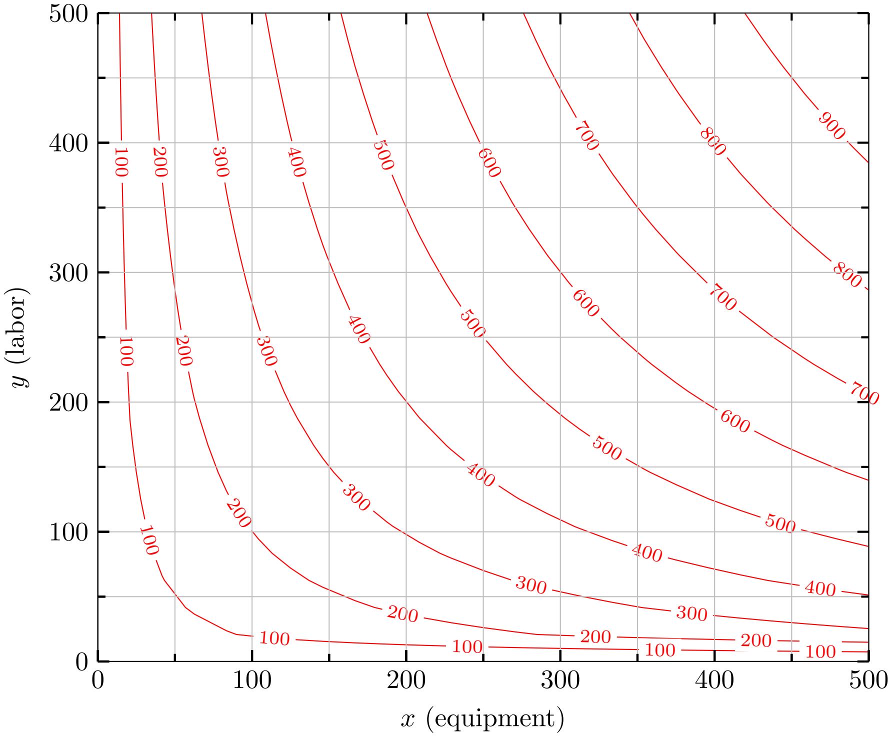
this parabola opens upward, so its minimum occurs at the vertex:

$$q = \frac{-(-12)}{2(1)} = 6.$$

Therefore the average cost is minimized when

$$\boxed{q = 6 \text{ tons}}.$$

11. A firm produces a single product with two inputs: equipment and labor. The contour plot below shows values of $f(x, y)$, the production level with x dollars in equipment and y dollars in labor.



(a) (6 points) Estimate $f(350, 100)$ and $f_y(350, 100)$.

Solution: From the contour plot, the point $(350, 100)$ lies very close to the contour labeled 400, so

$$\boxed{f(350, 100) \approx 400.}$$

To estimate $f_y(350, 100)$, look vertically near $x = 350$: the contours 400 and 500 occur at about $y \approx 100$ and $y \approx 160$, respectively. Thus increasing y by about 60 increases f by about 100, so

$$f_y(350, 100) \approx \frac{100}{60} \approx 1.7.$$

Hence

$$\boxed{f_y(350, 100) \approx 1.7 \text{ units of output per labor dollar.}}$$

(b) (4 points) Explain the meaning of $f(350, 100)$ and $f_y(350, 100)$ in a sentence with correct units.

Solution: $f(350, 100) \approx 400$ means that investing \$350 in equipment and \$100 in labor gives a production level of about 400 units. The value $f_y(350, 100) \approx 1.7$ means that, near that point, increasing labor spending by \$1 while keeping equipment fixed increases production by about 1.7 units.

(c) (6 points) Mark the **greater** quantity in each pair below.

12. Let $f(x, y) = (x + xy)^{10}$, and compute the following partial derivatives.

(a) (6 points) $f_x(x, y)$

Solution: Let $u = x + xy = x(1 + y)$. Then $f = u^{10}$, so by the chain rule,

$$f_x = 10u^9\, u_x = 10(x + xy)^9(1 + y).$$

Thus,

$$\boxed{f_x(x, y) = 10(x + xy)^9(1 + y).}$$

(b) (6 points) $f_{xx}(x, y)$

Solution: Differentiate $f_x = 10(x + xy)^9(1 + y)$ with respect to x:

$$f_{xx} = 10(1 + y) \cdot 9(x + xy)^8(1 + y).$$

So

$$\boxed{f_{xx}(x, y) = 90(x + xy)^8(1 + y)^2.}$$

(c) (6 points) $f_{xy}(x, y)$

Solution: Differentiate $f_x = 10(x + xy)^9(1 + y)$ with respect to y:

$$f_{xy} = 10\left[9(x + xy)^8(x)(1 + y) + (x + xy)^9\right].$$

Factoring gives

$$f_{xy} = 10(x + xy)^8\big(9x(1 + y) + (x + xy)\big).$$

Since $x + xy = x(1 + y)$,

$$9x(1 + y) + (x + xy) = 10x(1 + y).$$

Therefore,

$$\boxed{f_{xy}(x, y) = 100x(1 + y)(x + xy)^8.}$$

13. A firm manufactures two products with one priced at x dollars and one at y dollars. At prices x and y, the firm's combined revenue for the two products is

$$R = f(x, y) = -4x^2 - 4xy - 3y^2 + 100x + 130y$$

(a) (8 points) Find the first- and second-order partial derivatives f_x, f_y, f_{xx}, f_{yy} and f_{xy} of the revenue function.

Solution:

$$f_x = -8x - 4y + 100, \qquad f_y = -4x - 6y + 130.$$

Also,

$$f_{xx} = -8, \qquad f_{yy} = -6, \qquad f_{xy} = -4.$$

(b) (6 points) Use your partial derivatives to find any critical points for the revenue function. Show your work.

Solution: Set the first partial derivatives equal to zero:

$$-8x - 4y + 100 = 0, \qquad -4x - 6y + 130 = 0.$$

Simplifying,

$$2x + y = 25, \qquad 2x + 3y = 65.$$

Subtract the first equation from the second:

$$2y = 40 \implies y = 20.$$

Then

$$2x + 20 = 25 \implies x = \frac{5}{2}.$$

So the only critical point is

$$\boxed{\left(\frac{5}{2}, 20\right)}.$$

(c) (6 points) Find prices x and y that give the largest possible revenue. Include some computations that show that the revenue is maximized at these prices.

Solution: Use the second derivative test:

$$D = f_{xx}f_{yy} - (f_{xy})^2 = (-8)(-6) - (-4)^2 = 48 - 16 = 32 > 0.$$

Since $f_{xx} = -8 < 0$, the critical point is a local maximum. Because f is a quadratic function with negative definite Hessian, this is also the global maximum. Thus the revenue is maximized at

$$\boxed{x = \frac{5}{2} \text{ dollars}, \qquad y = 20 \text{ dollars}.}$$

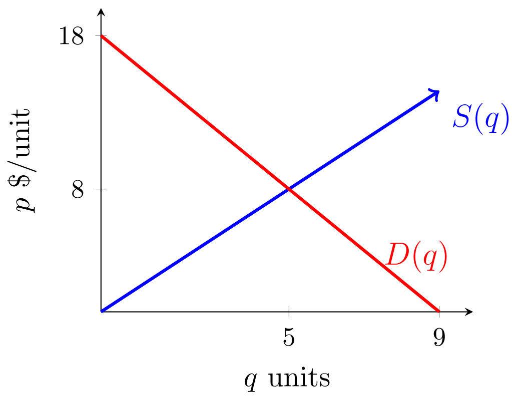
The maximum revenue is

$$f\left(\frac{5}{2}, 20\right) = -4\left(\frac{5}{2}\right)^2 - 4\left(\frac{5}{2}\right)(20) - 3(20)^2 + 100\left(\frac{5}{2}\right) + 130(20) = 1325.$$

So the largest possible revenue is

$$\boxed{1425}.$$

14. The graph below shows supply and demand graphs for a good.



(a) (6 points) Use geometry to find exact values for $\int_0^5 S(q)\,dq$ and $\int_0^5 D(q)\,dq$.

Solution: For $S(q) = \frac{8}{5}q$, the region under the graph from 0 to 5 is a triangle with base 5 and height 8, so

$$\int_0^5 S(q)\,dq = \frac{1}{2}(5)(8) = 20.$$

For $D(q) = 18 - 2q$, the region under the graph from 0 to 5 is a trapezoid with bases 18 and 8 and width 5, so

$$\int_0^5 D(q)\,dq = \frac{1}{2}(18+8)(5) = 65.$$

Therefore,

$$\boxed{\int_0^5 S(q)\,dq = 20, \qquad \int_0^5 D(q)\,dq = 65.}$$

(b) (4 points) What is the equilibrium price and quantity for this market?

$(p^*, q^*) = \boxed{(8, 5)}$

(c) (6 points) Compute the value of the consumer surplus.

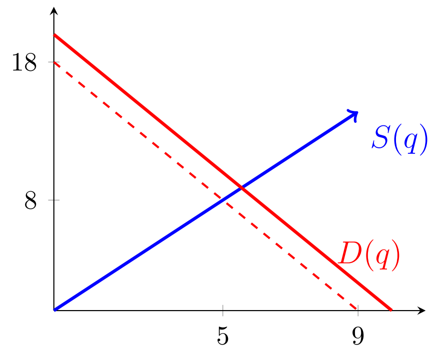
Solution: Consumer surplus is the area between the demand curve and the equilibrium price from $q = 0$ to $q = 5$:

$$CS = \int_0^5 D(q)\,dq - p^*q^* = 65 - (8)(5) = 25.$$

Thus,

$$\boxed{\text{consumer surplus} = 25.}$$

(d) (6 points) Suppose the demand curve moves from the position shown above to a new position below.



Mark all of the following statements that are correct. Compared the to original demand curve ...

$\surd$ $p*$ **will increase.**

$\surd$ $q*$ **will increase.**

$\surd$ **the consumer surplus will increase.**

Formulas

Derivatives:

- $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$
- $\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$
- $\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$
- $\frac{d}{dx}(c) = 0$, if c is a constant
- $\frac{d}{dx}(mx + b) = m$,
  where m, b are constants
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$
- $\frac{d}{dx}\left(a^x\right) = a^x \cdot \ln(a)$
- $\frac{d}{dx}\left(e^x\right) = e^x$
- $\frac{d}{dx}\left(e^{kx}\right) = k \cdot e^{kx}$, if k is a constant
- $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

Other things:

- Quadratic formula: $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Future and present value, yearly compounding: $FV = PV \cdot (1 + r)^t$
- Future and present value, continuously compounding: $FV = PV \cdot e^{rt}$
- Elasticity: $E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|$
- Consumer surplus: $\int_0^{q^*} D(q)\, dq - p^* q^*$
- Producer surplus: $p^* q^* - \int_0^{q^*} S(q)\, dq$
- Present value of income stream $S(t)$:

$$PV = \int_0^M S(t) e^{-rt}\, dt$$

- Future value of income stream $S(t)$:

$$FV = e^{rM} \cdot PV = \int_0^M S(t) e^{r(M-t)}\, dt$$

- Second derivative test:
  - Compute discriminant at critical points (a, b):

    $$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - (f_{xy}(a, b))^2$$

  - $D(a, b) > 0$ and $f_{xx}(a, b) > 0 \Rightarrow$ local min
  - $D(a, b) > 0$ and $f_{xx}(a, b) < 0 \Rightarrow$ local max
  - $D(a, b) < 0$ no local max/min (Saddle Point)
  - $D(a, b) = 0$ inconclusive test